Comment on "Thermodynamic transitions in inhomogeneous *d*-wave superconductors"

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Scanning-tunneling spectroscopy studies on $Bi_2Sr_2CaCu_2O_{8+\delta}$ suggest the presence of electronic inhomogeneity with a large spatial variation in gap size. Andersen *et al.* have modeled this variation by assuming a spatially varying pairing interaction. We show that their calculated specific heat is incompatible with the experimental data which exhibit narrow transitions. This calls into question the now-common assumption of gap and pairing inhomogeneity.

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In a recent paper, Andersen *et al.*[1] model the spectroscopic inhomogeneity inferred from scanning-tunneling spectroscopy (STS) studies on $Bi_2Sr_2CaCu_2O_{8+\delta}$ (Bi-2212).[2,3] These STS studies suggest a $\sim\pm25\%$ variation in the peak-to-peak gap magnitude on a length scale of a coherence length, ξ_0. The locations with large spectral gaps lack well-developed coherence peaks and, based on the absence of Ni resonances there, they were suggested to be nonsuperconducting.[3]

To model these effects Andersen *et al.*[1] adopt a spatially inhomogeneous pairing interaction in a *d*-wave BCS pairing model and solve self-consistently for the local gap magnitude in order to map the spatially varying interaction onto the observed variation in gap magnitude. They then compute the specific heat and show that the breadth of the anomaly is similar to the spread in local gap magnitude. This breadth, they claim, is "similar to experimental observations," and they conclude that "substantial nanoscale electronic inhomogeneity is characteristic of the bulk BSCCO system."

However, our data[4] (see Fig. 1), which they use for their comparison, suggest the opposite conclusion. The sharpness of each transition, indicated by the arrows in Fig. 1(a), actually precludes pairing and gap inhomogeneity on the scale envisaged by Andersen *et al.* They mistakenly equate the extended fluctuation region observed above T_c with broadening of their mean-field (MF) transition over a 40 K range. Fluctuations are an intrinsic property of this highly anisotropic material[5] but are not included in their model. In fact, it is the narrow region of strong negative curvature close to the top of each peak [see arrows in Fig. 1(a)] that reveals the true extent of transition broadening, in this case rather small. Such a strong T dependence over a very restricted T range would not be possible in a material with a broad distribution of T_c values.

There are other problems. The locations where their gap is a maximum correspond to the maximal local pairing interaction and the maximum local contribution to the condensation energy, i.e., where superconductivity is strongest. The Ni resonances in STS studies[3] show the opposite: superconductivity is weakest and perhaps absent at the points where the supposed gap is maximal (and the coherence peaks are absent). It is now becoming apparent that these large gaps are not superconducting gaps at all but the pseudogap near $(\pi,0)$.[6–8] The pseudogap, with its distinctive absence of coherence peaks, has recently been observed at these large-gap locations for $T\gg T_c$.[9]

Here we consider the specific-heat data in more detail and show that the transitions are not strongly broadened as suggested by Andersen *et al.*,[1] thus reversing their inference of inhomogeneous pairing in the bulk material.

The specific heat near T_c consists of a MF step at T_c and a (nearly symmetrical) fluctuation contribution above and below T_c.[5] More generally, T_c may be broadened out into a distribution of T_c values. The separate contributions of fluctuations and transition broadening may seem similar well away from the mean T_c, but nearby they are quite distinctive and easily separated.

Let us consider just the case of fluctuations where there is a sharply defined T_c. (Fluctuations in the presence of a distribution of T_c values is treated elsewhere.[10]) The fluctuation

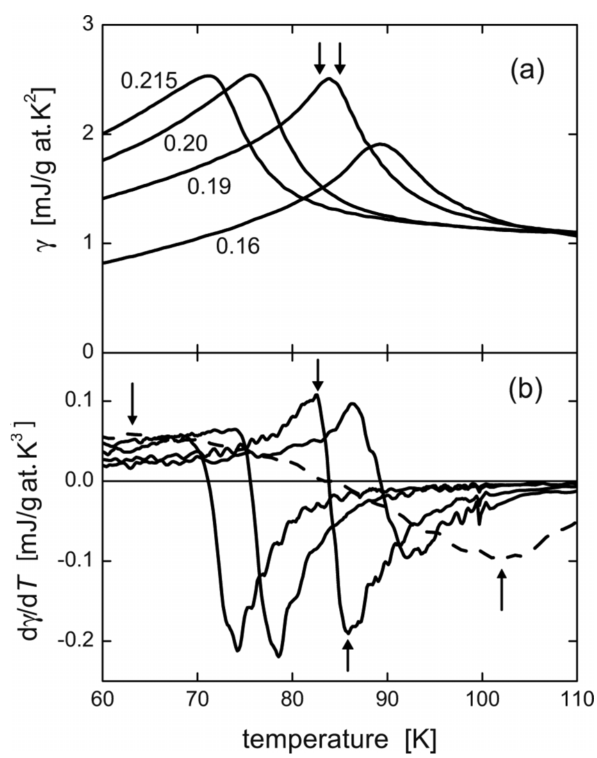

FIG. 1. (a) The specific-heat coefficient, γ, for Bi-2212 with $p=0.16$, 0.19, 0.20, and 0.215, respectively. (b) The derivative $\partial\gamma/\partial T$. The dashed curve is $\partial\gamma/\partial T$ from the Andersen *et al.* calculation. Arrows indicate inflection points in $\gamma(T)$.

specific heat should diverge at T_c but is cut off due to the inhomogeneity length scale. For Bi-2212 we have previously analyzed the fluctuation contribution[4] and deduced transition half-widths as small as $\Delta T_c/T_c \sim 0.014$, consistent with an inhomogeneity length scale as large as $16\xi_0$, much greater than the length ξ_0 suggested by STS.[2,3] The cutoff is reflected in the narrow region of negative curvature between the inflection points in the specific-heat coefficient, $\gamma(T)$ near T_c (arrows in Fig. 1).

In Fig. 1(b) we show the derivative $\partial\gamma/\partial T$ from our data and compare it with that from Andersen's model calculation. The data curves correspond to doping states of $p=0.16$, 0.19, 0.20, and 0.215. The inflection points are located at the maxima and minima below and above T_c, and between them $\partial\gamma/\partial T$ changes sign. For $p=0.19$ the inflection points are just 3.3 K apart. For Andersen's calculation[1] they are up to 40 K apart (dashed curve and arrows), just as would be expected for a $\pm 25\%$ spread of gap values. Evidently, the model calculations are not "similar to experimental observations" and by implication the inferred inhomogeneity is not sustainable.

The inclusion of fluctuations in their model is not expected to alter this. Based on theoretical work by Fisher and Barber[11] and by Thouless[12] the rounding of both the fluctuation and mean-field terms is approximated by the substitution $t \rightarrow [(t+\delta)^2+\Delta^2]^{1/2}$, where the shift, δ, and broadening, Δ, are determined by the inhomogeneity and field length scales L_0 and L_H, respectively. This illustrates the crucial point that fluctuation and mean-field terms are subject to the same degree of broadening.

Pertinent to this issue are the Monte Carlo calculations of Ebner and Stroud[13] on coupled granular materials which include both mean-field and fluctuation effects. They consider grains with the same local T_c with either identical or inhomogeneous coupling, and also coupled grains with a spread of local T_c values. The latter case most closely approximates to the Andersen model. An important conclusion from this paper is that for weak to moderate coupling strength the specific-heat transition is dominated by the peaks associated with isolated grains and shows the characteristic rounding due to finite-size effects and the spread of local T_c's of the individual grains. The authors state that this results because "the specific-heat peak mainly reflects amplitude degrees of freedom of the superconducting order parameter which turn on at the single-grain transition." There is a phase transition at a phase-ordering temperature which is usually lower than the specific-heat peak temperature. This barely shows up in the specific heat (even including fluctuation effects) because of the very small number of phase degrees of freedom that are involved. If the coupling is sufficiently strong the T_c's of individual grains become locked together and the specific-heat transition then approximates to that of a homogeneous superconductor.

In the model described in Andersen *et al.*, regions of high and low gap are interconnected through the Δ_{ij} and t_{ij} terms, so "coupling" effects are built in. This is responsible for the growth of superconducting islands as T_c is approached from above. In spite of this the specific-heat transition, which reflects this growth in the magnitude and extent of the order parameter, still shows considerable broadening which, as we have pointed out, far exceeds the experimental transition width. We reiterate that fluctuations associated with local phase ordering in the grains would have been broadened to the same extent as the mean-field step, and the same is true for any weak anomaly at the phase-ordering temperature.

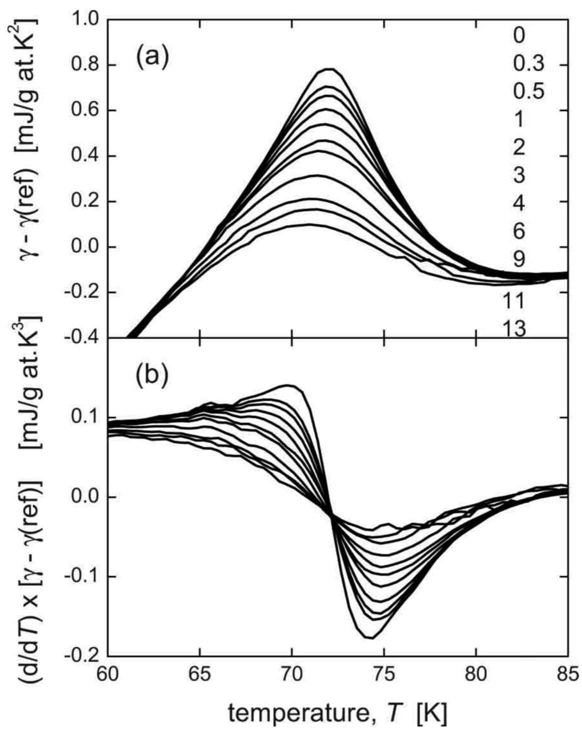

FIG. 2. (a) The field dependence of the specific-heat coefficient $\gamma-\gamma(\text{ref})$ for Bi-2212. The field is shown in units of tesla. (b) The derivative, $\frac{d}{dT}\times[\gamma-\gamma(\text{ref})]$, showing the field broadening of the inflection points.

To show that the transition is indeed narrow we plot in Fig. 2 the field dependence of γ for a Bi-2212 sample with doping $p\approx 0.21$. First it is clear from this plot that $T_c(H=0)$ is close to the peak (as expected if the MF step is small relative to the fluctuation term). The narrow peak is progressively suppressed and broadened by the field, with a marked effect even for fields as low as 0.3 T. The vortex separation $L_H \sim \sqrt{\phi_0/H} \sim 45\ \text{nm}/\sqrt{H(\text{tesla})}$, which acts as an inhomogeneity length scale, is very large at low field and the sensitivity of the transition to fields as low as 0.3 T supports our conclusion that the order parameter is rather homogeneous. Calculated transitions based on an inhomogeneity length scale of $\xi_0 \sim 2$ nm would be totally insensitive to such low fields.

Though, in a granular system, coupling between the grains will allow longer-range fluctuations on a scale exceeding L_0, the typical grain size, we reiterate that for weak to moderate coupling there is almost no feature at the phase-ordering temperature. Therefore, the effect of a magnetic field there would be extremely weak. The main specific-heat peak is characteristic of the individual grains and is broadened by a combination of the distribution of T_c values for the grains and finite-size effects expected for the grain size, L_0, for each individual grain. Our above argument therefore still applies, namely, that the additional broadening of the specific-heat peak by a weak magnetic field would not be detectible if $L_H \gg L_0 \approx \xi_0$.

To conclude, we have shown that the inference of pairing inhomogeneity from STS gap maps, and the resultant transition broadening, is inconsistent with the specific-heat data which exhibit sharp features with transition widths of the order of 3 K in Bi-2212. It is not possible with any broad spread of SC gaps to have strong T dependences in $\gamma(T)$ over such a narrow T range. Further, the inference of gross gap inhomogeneity does not appear to be supported by other STS data[14] and possibly just reflects scattering effects.[15]

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